TO'G'RI CHIZQDAGI KESMALARNING MINKOVSKIY AYIRMASI

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Annotatsiya: Ushbu ishda Minkovskiy operatori yordamida to'g'ri chiziqdagi kesmalar ustida amallarni bajarishning asosiy tushunchalari va xossalari o'rganiladi. Minkovskiy ayirmasi – geometriyada ko'p qo'llaniladigan tushuncha bo'lib, chiziqli fazoda kesmalar orasidagi munosabatlarni aniqlashda muhim vosita hisoblanadi. Ish davomida kesmalar ustidagi amallar, ularning geometrik va algebraik xossalari, shuningdek, Minkovskiy ayirmasi natijasida hosil bo'ladigan yangi kesmalar ko'rib chiqiladi. Tadqiqot, ayniqsa, kesmalarni amaliy masalalarda qo'llash, masalan, kompyuter grafikasi, robototexnika va mexanik tizimlar modellashida Minkovskiy operatorining rolini yoritishga qaratilgan

Kalit so'zlar: Minkovskiy ayirmasi, kesmalar, to'g'ri chiziq, geometrik xossalar, algebraik xossalar, fazoviy munosabatlar, kompyuter grafika, robototexnika, mexanik tizimlar.

Annotation. This work studies the fundamental concepts and properties of operations on line segments using the Minkowski operator. The Minkowski difference is a widely used concept in geometry, serving as an important tool for defining relationships between segments in linear spaces. The research explores operations on segments, their geometric and algebraic properties, as well as the new segments resulting from the Minkowski difference. The study particularly focuses on the application of segments in practical problems, such as the role of the Minkowski operator in computer graphics, robotics, and mechanical system modeling.

Keywords: Minkowski difference, segments, straight line, geometric properties, algebraic properties, spatial relationships, computer graphics, robotics, mechanical systems.

Аннотация. В данной работе изучаются основные концепции и свойства операций над отрезками прямой с использованием оператора Минковского. Разность Минковского является широко используемым понятием в геометрии и служит важным инструментом для определения взаимосвязей между отрезками в линейных пространствах. В исследовании рассматриваются операции с отрезками, их геометрические и алгебраические свойства, а также новые отрезки, полученные в результате разности Минковского. Особое внимание уделяется применению отрезков в практических задачах, таких как роль оператора Минковского в компьютерной графике, робототехнике и моделировании механических систем.

Ключевые слова: разность Минковского, отрезки, прямая, геометрические свойства, алгебраические свойства, пространственные отношения, компьютерная графика, робототехника, механические системы.

Minkovskiy amallari – ya'ni, to'plamlarning Minkovskiy yig'indisi va ayirmasi – matematikada geometriya va algebra sohalarida muhim ahamiyatga ega. Ular asosan qavariq to'plamlar va ko'pburchaklar bilan ishlashda qo'llaniladi va turli sohalarga, jumladan, robototexnika, kompyuter grafikasi, optimizatsiya va mexanika masalalariga tatbiqlari mavjud. Ushbu amallar oddiy geometrik shakllar bilan ishlashni osonlashtirib, ularning hajmi, yuzasi va boshqa o'lchovlarini aniqlashda samaradorlikni oshiradi.

Bu ishda to'g'ri chiziqdagi kesmalarnig Minkovskiy yig'indisi va ayirmasini topish olingan natijalar bayon etilgan. Dastlab Minkovskiy yig'indisi va ayirmasi amallarini aniqlab olamiz.

1-ta’rif. $\mathbf{R}^n$ fazoda bo‘sh bo‘lmagan $X,Y\subset\mathbf{R}^n$ to‘plamlar berilgan bo‘lsin. X va Y to‘plamlarning geometrik yig‘indisi va ayirmasi yoki Minkovskiy yig‘indisi va ayirmasi deb mos ravishda quyidagi to‘plamlarga aytiladi:

$$X+Y=\{x+y:x\in X,y\in Y\},\quad X\overset{*}{-}Y=\{z\in\mathbf{R}^n:z+Y\subset X\}.$$

Yuqoridagi ta’rifga ko‘ra ixtiyoriy $a\in\mathbf{R}^n$ vektor va bo‘sh bo‘lmagan $X\subset\mathbf{R}^n$ to‘plamning Minkovskiy yig‘indisi deganda

$$a+X=\{a+x:x\in X\}$$

to‘plamni tushinamiz.

To‘plamlar ustidagi amallarning bajarilish tartibi quyidagicha kelishib olingan. Qavs bo‘lmasa birinchi navbatda songa ko‘paytirish amali bajariladi, chapdan o‘nga qarab geometrik yig‘indi va geometrik ayirma amallaridan qaysi amal birinchi kelsa o‘sha amal bajariladi.

Shuningdek, bu maqolada $\mathbf{R}^n$ fazoda bo‘sh bo‘lmagan $X,Y,Z,X_1,$ X_2,Y_1,Y_2 to‘plamlar va ixtiyoriy $d_1,d_2\in\mathbf{R}^n$ vektorlar uchun Minkovskiy ayirmasi va yig‘indisining quyidagi xossalari keltirilgan:

1) $X+Y=Y+X$;
2) $X+(Y+Z)=X+Y+Z$;
3) $X\overset{*}{-}(Y+Z)=X\overset{*}{-}Y\overset{*}{-}Z$;
4) a) $\lambda(X+Y)=\lambda Y+\lambda X$;
   b) $\lambda(X\overset{*}{-}Y)=\lambda Y\overset{*}{-}\lambda X$;
   c) $(\lambda\mu)X=\lambda(\mu X)$;
5) $(X+d_1)\overset{*}{-}(Y+d_2)=X\overset{*}{-}Y+d_1-d_2$;
6) a) $X\overset{*}{-}Y+Y\subset X$;
   b) $X\subset X+Y\overset{*}{-}Y$;
   c) $Y\subset X\overset{*}{-}(X\overset{*}{-}Y)$;
7) a) agar $X\subset Y$ bo‘lsa, u holda $\lambda X\subset\lambda Y$ bo‘ladi;
   b) agar $X_1\subset X_2,Y_1\subset Y_2$ bo‘lsa, u holda $X_1+Y_1\subset X_2+Y_2$ bo‘ladi;
   c) agar $X_1\subset X_2,Y_1\supset Y_2$ bo‘lsa, u holda $X_1\overset{*}{-}Y_1\subset X_2\overset{*}{-}Y_2$ bo‘ladi;
8) a) $X+(Y\overset{*}{-}Z)\subset X+Y\overset{*}{-}Z$;
   b) $X+(Y\overset{*}{-}Z)\subset Y\overset{*}{-}(Z\overset{*}{-}X)$;
9) a) $X\overset{*}{-}Y+Y\overset{*}{-}Y=X\overset{*}{-}Y$;
   b) $X+Y\overset{*}{-}Y+Y=X+Y$;
   c) $X\overset{*}{-}(X\overset{*}{-}(X\overset{*}{-}Y))=X\overset{*}{-}Y$;
10) a) $(X\overset{*}{-}Y)^c=-Y+X^c$;
   b) $(X+Y)^c=X^c\overset{*}{-}(-Y)$;
   c) $X^c\overset{*}{-}Y^c=-(Y\overset{*}{-}X)$;

11) $Y \overset{*}{-} \left(Y \overset{*}{-} X\right) = X + \left(-Y\right)^{c} \overset{*}{-} \left(-Y\right)^{c}$;

12) a) $\left(X \overset{*}{-} Z\right) \cap \left(Y \overset{*}{-} Z\right) \subset \left(X \cap Y\right) \overset{*}{-} Z$;

b) $\left(X \overset{*}{-} Y\right) \cap \left(X \overset{*}{-} Z\right) \subset X \overset{*}{-} \left(Y \cup Z\right)$.

Bu yerda X^{c} - X to'plamning $\mathbf{R}^{n}$ fazoga to'ldiruvchisi, ya'ni $X^{c} = \left\{x \in \mathbf{R}^{n} : x \notin X\right\}$ va λ, μ - haqiqiy sonlar.

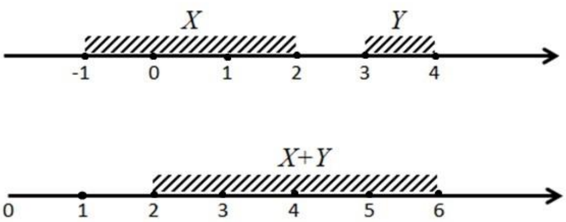
1-teorema. □ to'g'ri chiziqda berilgan $X = (a,b)$ va $Y = (a_1,b_1)$ intervallarning Minkovskiy yig'indisi $X + Y = (a + a_1, b + b_1)$ intervalga teng bo'ladi.

Isbot. Minkovskiy yig'indisining aniqlanishiga ko'ra (a,b) va (a_1,b_1) intervallarning Minkovskiy yig'indisi (a,b) intervalning har bir elementiga (a_1,b_1) intervalning har bir elementini qo'shish natijasida hosil bo'ladigan to'plamdir. Ixtiyoriy $x \in X$ ya'ni $a < x < b$ elementni barcha $y \in Y$ ya'ni $a_1 < y < a_2$ elementlarga qo'shib chiqamiz. U holda tengsizlik xossasiga ko'ra $a + a_1 < x + y < b + b_1$ munosabat o'rinli bo'ladi. x va y lar mos ravishda X va Y to'plamlarning ixtiyoriy elementlari bo'lganligidan $X + Y = (a + a_1, b + b_1)$ kelib chiqadi.

Xuddi shunday tarzda, □ to'g'ri chiziqda berilgan quyidagi ko'rinishdagi to'plamalarning Minkovskiy yig'indisini topish mumkin:

$$\begin{aligned} &[a,b] + [a_1,b_1] = [a + a_1, b + b_1]; \\ &[a,b] + (a_1,b_1) = (a + a_1, b + b_1); \\ &[a,b) + (a_1,b_1] = (a + a_1, b + b_1); \\ &\{a\} + (a_1,b_1) = (a + a_1, a + b_1). \end{aligned} \tag{1}$$

Misol: $X = (-1,2)$, $Y = (3,4)$ bo'lsa $X + Y = (2,6)$ bo'ladi (1-rasm).



1-rasm. To'g'ri chiziqda ikki to'plamning Minkovskiy yig'indisi

2-teorema. □ to'g'ri chiziqda berilgan $X = (a,b)$ va $Y = (a_1,b_1)$ intervallarning Minkovskiy ayirmasi uchun quyidagi munosabat o'rinli bo'ladi:

$$X \overset{*}{-} Y = \begin{cases} [a - a_1, b - b_1] & agar\ a - a_1 < b - b_1\ bo'lsa, \\ \{a - a_1\} & agar\ a - a_1 = b - b_1\ bo'lsa, \\ \varnothing & agar\ a - a_1 > b - b_1\ bo'lsa. \end{cases} \tag{2}$$

Isbot. Minkovskiy ayirmasining aniqlanishiga ko'ra har qanday $z \in X \overset{*}{-} Y$ uchun $z + Y \subset X$ bo'ladi. $z + Y \subset X$ munosabatdagi to'plamlarni mos intervallar bilan almashtiraylik, ya'ni $z + (a_1,b_1) \subset (a,b)$. 1-teoremaga ko'ra $(z + a_1, z + b_1) \subset (a,b)$ bundan esa $z + a_1 \geq a$, $z + b_1 \leq b$ va $a - a_1 \leq z \leq b - b_1$ kelib chiqadi.

Agar $a-a_1<b-b_1$ bo‘lsa $a-a_1\le z\le b-b_1$ munosabat o‘rinli, agar $a-a_1=b-b_1$ bo‘lsa $z=a-a_1$ bo‘lishini ko‘rish qiyin emas.

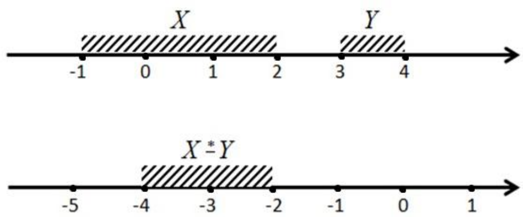
Agar $a-a_1>b-b_1$ bo‘lsa $a-a_1\le z\le b-b_1$ tengsizlikka ziddiyat hosil bo‘ladi va bu ziddiyatni qanoatlantiruvchi birorta ham $z\in X\overset{*}{-}Y$ element topilmaydi, ya’ni $X\overset{*}{-}Y=\varnothing$ bo‘ladi.

Xuddi shunday tarzda, to‘g‘ri chiziqda berilgan quyidagi ko‘rinishdagi to‘plamalarning Minkovskiy ayirmasini topish mumkin:

$$[a,b]\overset{*}{-}[a_1,b_1]=[a-a_1,b-b_1];$$
$$[a,b]\overset{*}{-}(a_1,b_1)=[a-a_1,b-b_1]; \quad (3)$$
$$(a,b)\overset{*}{-}[a_1,b_1]=(a-a_1,b-b_1);$$

bu formulalardagi Minkovskiy ayirmalari $a-a_1<b-b_1$ hol uchun hisoblangan.

Misol: $X=(-1,2),\ \ Y=(3,4)$ bo‘lsa $X\overset{*}{-}Y=[-4,-2]$ bo‘ladi (2-rasm).



2-rasm. To‘g‘ri chiziqda ikki to‘plamning Minkovskiy ayirmasi

Minkovskiy amallari nazariyasi zamonaviy matematikada keng qo‘llaniladigan, ko'p tarmoqli soha hisoblanadi. Bu amallar nafaqat sof matematikada, balki amaliy fanlar, jumladan, fizika, robototexnika, kompyuter grafika va tasvirlarni qayta ishlashda ham qo'llaniladi. Yig'indilar va farqlar orqali geometrik shakllarning harakatini, ularning bir-biriga ta'sirini tushunish mumkin. Tadqiqotlar shuni ko'rsatmoqdaki, Minkovskiy operatorlari yordamida murakkab masalalarni hal qilish va ularning matematik modellarini tuzish ancha osonlashadi. Demak, bu amallarni yanada chuqur o'rganish va ularning tadbiqlarini kengaytirish dolzarb masalalardan biri bo'lib qolmoqda.

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