CONSTRUCTION OF THE CENTER-VALUED QUASITRACES ON A FINITE REAL AW $*$-ALGEBRAS

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Abstract: In this thesis real AW*-algebras are considered. It is proved the existence and uniqueness of the centervalued quasitrace on finite real AW*-algebras in analog to the center-valued trace on finite real von Neumann algebras.

Key words: C*-algebras, factor, real and complex AW*- algebras; center-valued dimension function, center-valued quasitraces.

It is necessary to remind some concepts to understand the problem in this thesis. Let B(H) be the algebra of all bounded linear operators on a complex Hilbert space H. A weakly closed *-subalgebra M containing the identity operator 𝟙 in B(H) is called a W*- algebra.

A weakly closed *-subalgebra $R \subset B(H)$ with the 𝟙 identity is called a real W*-algebra, if it is weakly closed and $R \cap iR = \{0\}$. A C*-algebra is a (complex) Banach *-algebra whose norm satisfies the identity $||x * x|| = ||x||^2$. Now let A be a real Banach *-algebra. A is called a real C*-algebra, if $A_c = A + iA$ can be normed to become a (complex) C*-algebra by extending the original norm on A. Note that a C*-norm on A_c is unique, if it exists. It is known that [1] A is real C*-algebra if and only if $||x * x|| = ||x||^2$ and $1 + x * x$ is invertible, for any $x \in A$.

To motivate the next definitions, suppose A is a *-ring with unity, and let w be a partial isometry in A. If $e = w * w$, it results from $w = ww * w$ that $wy = 0$ iff $ey = 0$ iff $(1 - e)y = y$ iff $y \in (1 - e)A$, thus the elements that right-annihilate w form a principal right ideal generated by a projection. If S is a nonempty subset of A, we write $R(S) = \{x \in A: sx = 0, \text{ for all } s \in S\}$ and call R(S) the right-annihilator of S. Similarly, the set $L(S) = \{x \in A: xs = 0, \text{ for all } s \in S\}$ denotes the left-annihilator of S. A Baer *-ring is a *-ring A such that, for every nonempty subset S of A, $R(S) = gA$ for a suitable projection g. It follows that $L(S) = (R(S *)) *= (hA) *= Ah$ for a suitable projection h. A real (resp. complex) AW*-algebra is a real (resp. complex) C*-algebra that is a Baer *-ring (for more details see [2]).

It is known that, every W*-algebra is an AW*-algebra. The converse of it was shown to be false by J.Dixmier, who showed that exist commutative AW*-algebras

that cannot be represented (*-isomorphically) as W*-algebras on any Hilbert space.

An W*- or AW*- algebra is called a factor if its center is trivial. It is known that investigation of general W*-algebras can be reduced to the case of W*-factors, which are classified into types I, II and III. More precisely, any W*-algebra has a unique decomposition along its center into the direct sum of W*-factors of the I_{fin}, I_∞, II_1, II_∞ and III types. Similarly, AW*-factors are classified by types I, II and III and any AW*-algebra also has a unique decomposition along its center into the direct sum of AW*-factors of the I_{fin}, I_∞, II_1, II_∞ and III types.

Sure, we will now provide a definition of quasitraces.

Definition [3]. Let A be a unital C*-algebra. A 1-quasitrace τ on A is a function $\tau: A \longrightarrow \mathbb{C}$ which satisfies:

1. $\tau(x^*x)= (xx^*) \geq 0, x \in A$;
2. $\tau(a+ib)=\tau(a)+i\tau(b)$, for $a,b \in A_{sa}$, where $A_{sa}=\{a \in A: a^*=a\}$;
3. τ is linear on every abelian C*-subalgebra B of A;

Furthermore, τ is called a n-quasitrace ($n \geq 2$) if there exists a 1-quasitrace τ_n on $M_n(A) = A \otimes M_n(\mathbb{C})$ such that

4. $\tau(x)=\tau_n(x \otimes e_{11}), x \in A$,

where $(e_{ij})_{i,j=1}^{n}$ denote the matrix units of $M_n(\mathbb{C})$ and A_{sa} self-adjoint part of algebra A.

Based on the definition given above and the center-valued dimension function and center-valued quasitraces construction steps in [4] for C* algebras, we construct a center-valued quasitraces for the real C* algebra. For this, condition 2 given in Theorem 1.27 (on the construction of center-valued quasitraces in finite AW* algebras) requires the selection of element x in the real AW* algebra as follows:

$$x=\frac{x+x^*}{2}+\frac{x-x^*}{2}.$$

First, a quasisled is defined for unit real C* algebra through the selected element x, then the center-valued dimension function is defined for the finite real AW*-algebras, and a center-valued quasitrace is constructed.

Reference:

1. Ayupov Sh. A. and Rakhimov A. A., Real W*-algebras, Actions of groups and Index theory for real factors, VDM Publishing House Ltd. Beau-Bassin, Mauritius. (2010), 138p.
2. Berberian S. K., Baer *-rings, Grundlehren der mathematischen Wissenschaften 195, Springer-Verlag Berlin Heidelberg. (2011), 309p.
3. Haagerup U. Quasitraces on exact C*-algebras are traces. C. R. Math. Acad. Sci. Soc. R. Can. 36(2-3), 67–92 (2014)
4. Fehlker F, Quasitrace abd AW*-bundles, Monster, 20–25 (2018)
5. Boltaev K. K., qizi Azimova T. E. Description of Real AW*-Factors of Type I //EUROPEAN JOURNAL OF INNOVATION IN NONFORMAL EDUCATION. – 2022. – T. 2. – №. 2. – C. 413-421.
6. RAKHIMOV, A., RAXMONOVA, N., & SALLEH, Z. (2023). A CONNECTION BETWEEN RICKART REAL C*-ALGEBRA AND ENVELOPING RICKART C*-ALGEBRA.
7. Kamoldinovna, S. Y. (2023). TURISTIK MAHSULOTLAR VA XIZMATLARNI DIVERSIFIKATSIYALASHNI BOSHQARISH MEXANIZMLARINI TAKOMILLASHTIRISH. QO'QON UNIVERSITETI XABARNOMASI, 8, 66-69
8. Sultonova, Y. (2023). TURISTIK XIZMATLAR VA MAHSULOTLARNI DIVERSIFIKATSIYALASHNI BOSHQARISHNING XORIJ TAJRIBALARI VA UNDAN MAMLAKATIMIZ AMALIYOTIDA FOYDALANISH USLUBIYOTLARI. Iqtisodiyot Va ta'lim, 24(4), 318–326.
9. qizi Azimova T. E. ECONOMIC DIRECTIONS IN TEACHING MATHEMATICS //Intent Research Scientific Journal. – 2023. – T. 2. – №. 4. – C. 54-56.
10. Tojiyeva, M. M. (2022). BIZNESNI RIVOJLANTIRISH SAMARADORLIGI HAMDA UNI BAXOLASHNING USLUBIY YONDASHUVLARI. QO 'QON UNIVERSITETI XABARNOMASI, 5, 52-58.

11. Tojiyeva, M. M. Q., & Abdullayev, A. A. U. (2021). The use of modern technologies in statistical data collection. Asian Journal of Multidimensional Research, 10(12), 752-757.

12. Raxmonova, V. (2023). THE ROLE AND PLACE OF MATHEMATICAL MODELS IN TEACHING STUDENTS TO SOLVE OPTIMIZATION PROBLEMS. Modern Science and Research, 2(4), 592-597.