AYRIM DIFFERENSIAL TENGLAMALARNI WOLFRAM MATHEMATICA DASTURIDA YECHISH

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Annotatsiya: Mazkur tezisda ayrim chiziqli oddiy differensial tenglamalarni Wolfram Mathematica dasturidan foydalanib yechildi. Integral egri chiziqlarning grafiklari konstantaning ma’lum qiymatlari uchun yasab ko‘rsatildi. Shu bilan birga chiziqsiz differensial tenglamalarni yechishga doir misoldan namuna keltirildi.

Kalit so’zlar: Wolfram Mathematica, DSolve, Tabel, Plot, Show, Integrate, VectorPlot, differensial tenglama, integral egri chiziq, yechim, yo‘nalishlar maydoni.

Kompyuter matematikasi dasturlari (Maple, Mathematica, MatLab, Derive va boshqalar) fanning turli sohalarida qo‘llaniladi. Ular raqamli va analitik hisob-kitoblar, dasturlash vositalari va vizualizatsiya uchun protseduralarni o‘z ichiga oladi. Hozirgi vaqtda amaliy dasturlar paketlari faqat sonli masalalarni yechish uchun emas, balki teoremalarni isbotlash uchun ham qo‘llaniladi.

Wolfram Mathematica kompyuter dasturi Amerikaning Wolfram Research Inc kompaniyasi tomonidan ishlab chiqilgan bo‘lib, raqamli va simvolik hisoblashlarni juda samarali bajarish imkonini beruvchi eng keng tarqalgan dasturiy vositalardan biridir. U ikki o‘lchovli va uch o‘lchovli grafikalarga, shuningdek, o‘rnatilgan yuqori darajadagi dasturlash tiliga ega.

Wolfram Mathematica dastur paketi differensial tenglama yoki differensial tenglamalar sistemasini simvolik hamda sonli yechish imkonini beradi. Bundan tashqari, olingan natijalarni tasavvur qilish mumkin. Tabiiyki, har qanday matematik muammo birinchi navbatda fundamental bilimlar nazariyasining tahlil va sintez, induksiya va deduktsiya kabi klassik usullari bilan o‘rganiladi, so‘ngra uni rasmiylashtirish va algoritmik amalga oshirish sodir bo‘ladi. Wolfram Mathematica dasturlar paketida dasturlash tilining mavjudligi bizga dastlabki ma‘lumotlarni erkin o‘zgartirish mumkin bo‘lgan va ilgari surilgan gipotezalarni tasdiqlovchi yoki rad etuvchi keng ko‘lamli tajribalar o‘tkazish mumkin bo‘lgan keng toifadagi muammolar uchun dasturlar yozish imkonini beradi. Bu haqiqiy fizik jarayonlarni bashorat qilish uchun ularni o‘rganish imkonini beradi.

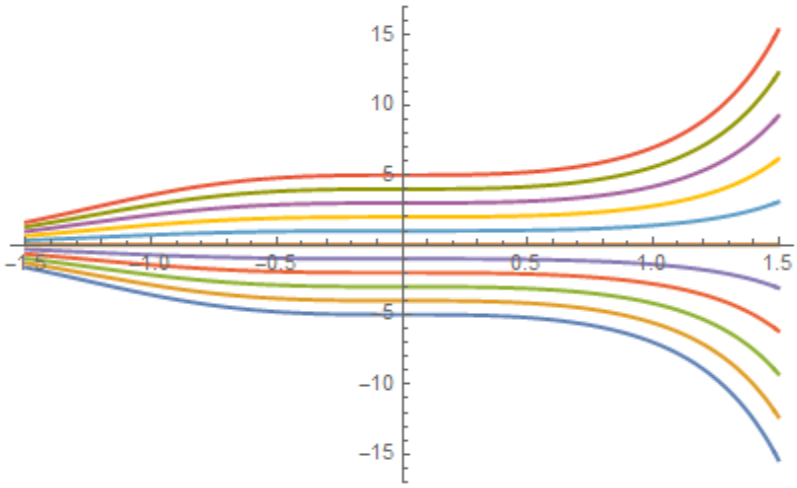
Geometrik masalalarni yechishda Wolfram Mathematica dasturidan foydalanishga misollar [1-5] adabiyotlarda keltirilgan. Shuningdek, [6-10] ishlarda ushbu dasturdan yechish murakkab bo‘lgan oddiy differensial tenglama uchun Grin funksiyasini tuzish jarayonida juda murakkab bo‘lgan to‘rtinchi tartibli determinantning qiymatini hisoblash uchun foydalanilgan.

Wolfram Mathematica oddiy differensial tenglamalar va ularning sistemalarini simvolik shaklda yechish uchun keng imkoniyatlarga ega. Buning uchun algoritmi hozirda ma‘lum bo‘lgan analitik usullarning ko‘pchiligini amalga oshiradigan **DSolve** buyrug‘idan foydalaniladi.

1-misol. $y' - x^2 y = 0$ differensial tenglamani yechamiz hamda integral egri chiziqlari oilasining grafiklarini o‘zgarmasning turli qiymatlari uchun tuzamiz.

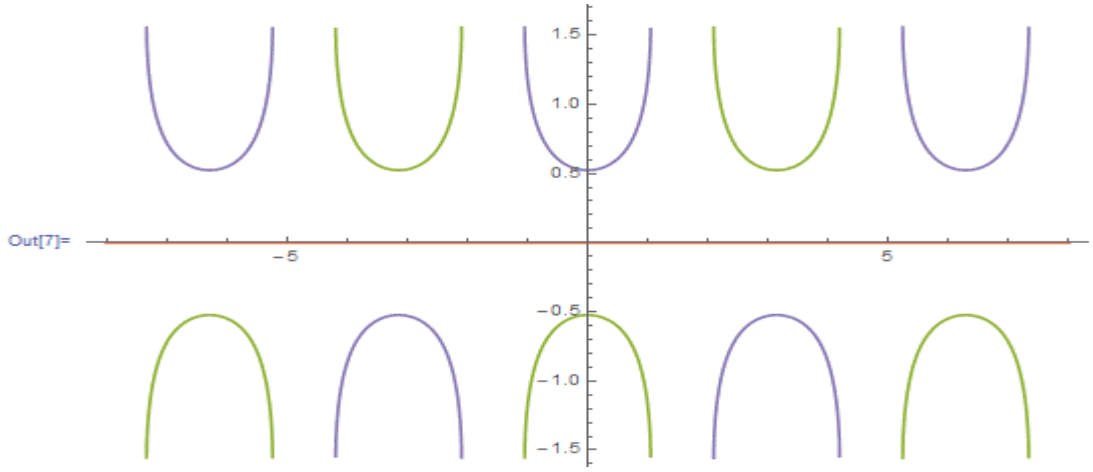
```
In[1]:= DS = DSolve[{y'[x] == x^2 y[x]}, y[x], x]

Out[1]= {{y[x] → e^(x^3/3) C[1]}}

In[2]:= tab = Table[y[x] /. DS[[1]] /. {C[1] → k}, {k, -5, 5, 1}]

Out[2]= {-5 e^(x^3/3), -4 e^(x^3/3), -3 e^(x^3/3), -2 e^(x^3/3), -e^(x^3/3), 0, e^(x^3/3), 2 e^(x^3/3), 3 e^(x^3/3), 4 e^(x^3/3), 5 e^(x^3/3)}

In[3]:= Plot[Evaluate[tab], {x, -1.5, 1.5}, PlotStyle → {Thickness[0.005]}]
```

Out[3]=



2-misol. $y' = tgx \cdot tgy$ differensial tenglama yechilsin va integral egri chiziqlar oilasining grafiklari konstantaning chizilsin.

```
In[5]:= DSolve[y'[x] == Tan[x] Tan[y[x]], y[x], x]

Solve::ifun : Inverse functions are being used by Solve,
    so some solutions may not be found; use Reduce for complete solution information. >>

Out[5]= {{y[x] → ArcSin[1/2 C[1] Sec[x]]}}

In[7]:= Plot[{-ArcSin[3/2 Sec[x]], -ArcSin[Sec[x]], -ArcSin[1/2 Sec[x]], 0,
    ArcSin[1/2 Sec[x]], ArcSin[Sec[x]], ArcSin[3/2 Sec[x]]}, {x, -8, 8}]
```



3-misol. $y'y^3 \sin y = \dfrac{x+9}{x^3+2}$ chiziqli bo‘lmagan differensial tenglamani yechamiz.

```
In[1]:= eq1 = y'[x] == (x + 9) / (y[x]^3 Sin[y[x]] (x^3 + 2))
```

$$\text{Out[1]= } y'[x] == \frac{(9+x)\,\text{Csc}[y[x]]}{(2+x^3)\,y[x]^3}$$

```
In[2]:= DSolve[eq1, y[x], x]
```

$$\text{Out[2]= } \left\{\left\{y[x] \to \text{InverseFunction}\left[-\text{Cos}[\#1]\,\#1\,\left(-6+\#1^2\right)+3\,\text{Sin}[\#1]\,\left(-2+\#1^2\right)\&\right]\left[C[1]+\frac{1}{12\times 2^{1/3}}\left(2\sqrt{3}\,\left(2+9\times 2^{2/3}\right)\text{ArcTan}\left[\frac{-1+2^{2/3}x}{\sqrt{3}}\right]+\left(-2+9\times 2^{2/3}\right)\left(2\,\text{Log}\left[2+2^{2/3}x\right]-\text{Log}\left[2-2^{2/3}x+2^{1/3}x^2\right]\right)\right)\right]\right\}\right\}$$

Ko‘rib turganingizdek, bu holda DSolve buyrug‘i yuqoridagi chiziqsiz tenglamani yecha olmadi. Shuning uchun, biz tenglamani

$$y^3 \sin y dy = \frac{x+9}{x^3+2}dx$$

shaklda yozib olamiz hamda har ikki tomonini integrallaymiz.

```
In[3]:= chapt = y^3 Sin[y]
```

$$\text{Out[3]= } y^3\,\text{Sin}[y]$$

```
In[5]:= ongt = (x + 9) / (x^3 + 2)
```

$$\text{Out[5]= } \frac{9+x}{2+x^3}$$

```
In[6]:= Integrate[chapt, y]
```

$$\text{Out[6]= } -y\left(-6+y^2\right)\text{Cos}[y]+3\left(-2+y^2\right)\text{Sin}[y]$$

```
In[7]:= Integrate[ongt, x]
```

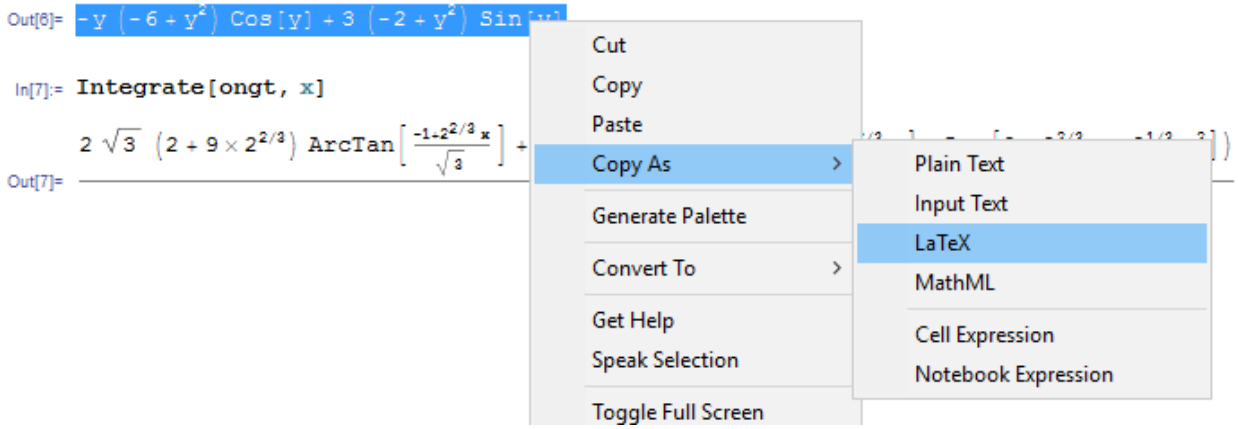
$$\text{Out[7]= } \frac{2\sqrt{3}\,\left(2+9\times 2^{2/3}\right)\text{ArcTan}\left[\frac{-1+2^{2/3}x}{\sqrt{3}}\right]+\left(-2+9\times 2^{2/3}\right)\left(2\,\text{Log}\left[2+2^{2/3}x\right]-\text{Log}\left[2-2^{2/3}x+2^{1/3}x^2\right]\right)}{12\times 2^{1/3}}$$

Shunday qilib, tenglamaning umumiy yechimi

$$3\left(y^2-2\right)\sin y - y\left(y^2-6\right)\cos y = \frac{2\sqrt{3}\left(2+9\sqrt[3]{4}\right)}{12\sqrt[3]{2}} ctg\left(\frac{\sqrt[3]{4}x-1}{\sqrt{3}}\right) +$$

$$+\frac{9\sqrt[3]{4}-2}{12\sqrt[3]{2}}\left(2\ln\left(\sqrt[3]{4}x+2\right)-\ln\left(\sqrt[3]{2}x^2-\sqrt[3]{4}x+2\right)\right)$$

shaklni oladi. Ko‘rib turganingizdek natija ham ancha murakkab ko‘rinishga ega bo‘lib, bu kabi tenglamalarni Wolfram Mathematica dasturi bizga osonlik bilan yechib beradi. Natijalarni matematik formulalarni yozuvchi dastur bo‘lgan MathType dasturiga nusxalash uchun bizga kerak bo‘lgan qismni belgilaymiz va kontekst menyudan CopyAs–>LaTex buyrug‘ini tanlaymiz hamda MathType dasturiga kirib, xotiraga olgan formulani joylab olamiz:



4-misol. $y'+2y-e^{-x}=0$ differensial tenglamani yechamiz va konstantaning turli qiymatlari uchun tenglama yechimining yo‘nalishlar maydoni hamda grafigini tuzamiz.

```
In[1]:= yechim = DSolve[y'[x] + 2 y[x] - E^(-x) == 0, y[x], x]
Out[1]= {{y[x] → e^-x + e^-2x C[1]}}
```

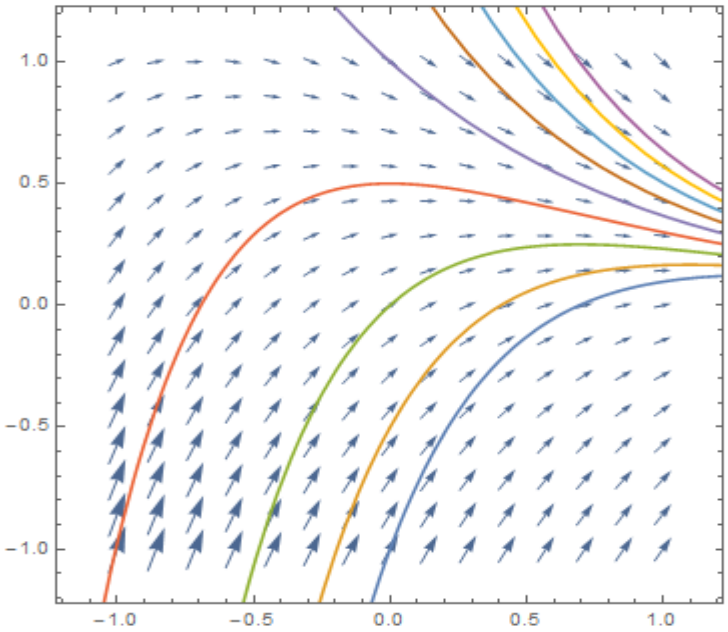
Endi C[1] ni *a* bilan almashtirib, yechimlar jadvalini tuzamiz, bunda *a* 0,5 qadam bilan -2 dan 2 gacha o‘zgaradi:

```
In[2]:= tab = Table[yechim[[1, 1, 2]] /. C[1] → a, {a, -2, 2, 0.5}]
Out[2]= {-2. e^-2x + e^-x, -1.5 e^-2x + e^-x, -1. e^-2x + e^-x, -0.5 e^-2x + e^-x,
         0. + e^-x, 0.5 e^-2x + e^-x, 1. e^-2x + e^-x, 1.5 e^-2x + e^-x, 2. e^-2x + e^-x}
```

Bir vaqtning o‘zida ikkita grafikni yasaymiz va yo‘nalishlar maydoni vektorlari differensial tenglama yechimlariga urinishini ko‘rsatamiz:

```
In[16]:= Show[VectorPlot[{1.5, E^(-x) - 2 y}, {x, -1, 1}, {y, -1, 1}],
          Plot[Evaluate[tab], {x, -2, 2}]]
```

Out[16]=



Xulosa

Wolfram Mathematica dasturining oddiy differensial tenglamalarga tatbiqlari haqida qisqacha ma’lumotlar keltirildi. Biz, asosan, DSolve, Tabel, Plot, Show, Integrate, VectorPlot hamda Evaluate kabi buyruqlardan foydalanishga doir misollardan namunalar keltirdik. Wolfram Mathematica dasturining yadrosi (Kernel) juda kuchli bo‘lib, o‘ta murakkab operatsiyalarni ham bajara oladi. Bu borada ushbu dastur yetakchilardan biridir. Hozirda Wolfram Mathematica dasturidan ko‘plab matematik olimlar o‘z ilmiy tadqiqotlarida foydalanayotganligini alohida ta’kidlab o‘tish joiz. Ushbu dasturni mukammal o‘rganish esa avvallari yechimini topish murakkab bo‘lgan ko‘plab masalalarni hal qilish imkonini berishiga ishonamiz.

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